DOCUMENT RESUME

ED 310 920 SE 050 858

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TITLE The Role of Children's Diagrams and Pictures in Their Solution of Complex Arithmetic Problems.
PUB DATE 89
NOTE 11p.; Paper presented at the Biennial Meeting of the Society for Research in Child Development (Kansas City, MO, April, 1989).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Arithmetic; *Diagrams; *Elementary School Mathematics; Foreign Countries; Grade 5; Grade 6; Intermediate Grades; Mathematical Applications; Mathematics Education; *Mathematics Skills; Mathematics Tests; *Problem Solving
IDENTIFIERS Canada; *Problem Solving Assessment

ABSTRACT

This research was concerned with the clarification of the role that the external representation process (e.g. drawing) plays in children's solving of complex arithmetic problems. Fourteen children in the 10- to 12-year-old age range were asked to represent and solve five complex arithmetic problems (three with temporal organization of data, two with spatial organization). The analysis of the representation process in relation to problem solving revealed that: (1) the older children had more ability to represent and to illustrate the relations between the data of the problem; and (2) discursive representation (reformulation of the problem by the child) seemed easier in the problems that called into play a temporal organization. The significance of these results for research in the area of problem solving in the teaching of mathematics is discussed. (Author/YP)

The role of children's diagrams and pictures in their solution of complex arithmetic problems

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Poster presented at the Biennial Meetings of the
Society for Research in Child Development, Kansas City, MO.
April, 1989

ABSTRACT

Recent studies have attempted to identify external representations (drawing, pictures...) that help children in solving problems. Difficulties encountered by children in the interpretation of these external representations, and the fact that they do not use them to solve arithmetic problems, have led us to ask whether is would not be preferable to investigate how students choose and then work with their own representations during the process of problem resolution.

Our research is concerned with the clarification of the role that this external representation process plays in children's solving of complex arithmetic problems.

Children in the 10 to 12 year old age range were asked to represent and solve complex arithmetic problems. The analysis of the representation process in relation with problem solving revealed that the role of this activity of external representation is closely related to the grade level and to the type of problem presented.

The significance of these results for research in the area of problem solving in the teaching of mathematics will be discussed.

AIMS

A number of teachers take for granted that the activity of external representation of a problem (that may take several forms: discursive, graphic,...) constitutes a support in problem solving activity. On the other hand, there is little confirming this evidence, this aspect having been little studied.

Our study seeks to:

Better grasp the contribution that the activity of external representation brings to problem solving, for certain complex arithmetic problems.

And clarify the function that this activity of external representation plays for the child within the process of problem resolution itself.

Research Questions:

1. When a problem is posed to a child to be solved, is the activity of external representation spontaneous or not? If spontaneous, to what elements is this activity related: grade level of the child, problem solving skills, type of problem presented?

2. Does the activity of external representation help in problem solving? If this activity constitutes a support in problem solving, for what children is it a support? (related to the ability level of the child)?

3. For a child, what role does this activity of external representation play within the process of problem resolution itself? Is this role the same for children of different grade level?

METHOD AND SUBJECTS

Reference framework underlying arithmetic problems presented to children

A reference framework (Denis, 1982; Carroll, Thomas et Malhorta, 1980) was used to construct a set of complex arithmetic problems, based on a conceptual analysis taking into account the following variables:

- *Type of relation between data*. The problems selected can be regrouped in two general types: those that call for a *temporal organization* and those that call for a *spatial organization*.

- *Evocation of images*. Some of the problem lend themselves more easily to constructing *mental images* than others.

Population

A heterogenous sample of 14 children was selected for each of the two grades (5th and 6th grade, 10 years old to 12 years old). These children attend a regular elementary school in a middle class neighborhood in a suburb of Montreal.

Selection of subjects

Each sample group was composed of children of different ability levels in problem solving (4 able, 4 average and 6 weak), selected by a preliminary written task.

Three aspects were taken into account in the characterization of the skills in problem solving of the children, based on previous research in this area (Greeno, 1980; Mayer, 1983): conceptual, linguistic and procedural knowledge.

The written task, touching these different elements, was composed of items on:

- solving complex arithmetic problems
- correction of problems
  (with missing, superfluous or non-related facts)
- formulation of problems (for given constraints)

The experiment: strutured individual interviews

Each child was given a set of complex arithmetic problems selected from a previous bank of problems (Bednarz, Janvier, Poirier, Biron, 1986), and based on the reference framework presented above.

Some of the problems were common to each grade level (5th and 6th) while others were common to both the preliminary written task and the interview.

TASK

Protocol

Five complex arithmetic problems were presented to children in grades five and six (3 with temporal organization of data, 2 with spatial organization). These problems were randomly ordered, and presented to child one at a time. Each child was interviewed individually outside the classroom. Each interview was videotaped.

For each problem, the protocol includes several steps:

- reading of the problem by the interviewer
- reformulation of the problem by the child. (This reformulation is the first index of the representation that the child constructs of the relation between the data of the problem).
- process of resolution after another reading of the problem by the interviewer (we note there the kind of spontaneous external representation used by the child to solve the problem).
- process of external representation, (this process of representation was provoked by the interviewer).

- process of resolution (once again the child was asked to solve the problem).

RESULTS

1. The activity of external representation appears spontaneously more frequently in grade 6 than in grade 5 (see table II). The type of problem presented is here determinant (this activity is clearly related to the variable "reference to mental images").

2. The reformulation of the problem (that take place between the written task and the first resolution of the same problem during the interview) seems to play an important role in the representation of the relations between the facts of the problem (see figure 2). This contribution of the activity of reformulation of the problem (table III) to the resolution of the problem appears to be especially determinant for the weaker children (skill in problem solving) (cf. table III).

3. The 6th grade chilren (cf. figure 3) have a greater tendency to represent the structure of the problem (they represent correctly the relations between the facts of the problem). In this activity of representation again, the type of problem appears determinant: the problems that most appear to evoke a correct representation are the problems that call for a temporal organization, and refer to mental images.

CONCLUSIONS

1. The role of the external representation seems to be closely related to the age of the problem solver and to the type of problem presented:

   - the older children (6th grade) have greater recourse to representation to illustrate the relations between the data of the problem, and relations are at this grade level more correctly represented.

- the discursive representation (reformulation of the problem by the child) seems easier in the problems that call into play a temporal organization. Among these, the problems invoking a mental image are more easily represented (schemas, illustrations...).

2. The activity of external representation (in particular the activity of reformulation) seems to play a role in problem solving, particularly for the weaker children.

Pedagogical consequences

The recourse to formulation, and reformulation of problems by pupils is a tool that could prove to be interesting as a learning strategy in problem solving.

A number of teachers take for granted that the activity of pictorial representation of a problem constitutes a support to understand and solve the problem. The results here show that we need to take into consideration a number of factors in this activity of representation of the problem (type of problem, age...).

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TABLE I

Problem structure	Examples of problems given to children	Other variable
Temporal organization	Problem 1 Alain and his friends are playing a game in which they can win or lose counters. The game has been going on for a while and it is now Alain's turn again. He plays, loses 7 counters and is allowed an immediate other trial. Knowing that he has now 3 counters more than prior to the last two trials, tell me if Alain won or lost counters at his second trial and how many? Problem - given in grades: 5 and 6	No reference to mental images.
Spatial organization	Problem 2 In a certain restaurant, we have a choice of a meat dish or a fish dish. With the fish dish we have a choice between 2 kinds of vegetables: mushrooms or mashed potatoes. With the meat dish we have a choice between 3 kinds of vegetables: french fries, carrots or turnips. For desert, we have a choice of ice cream or fruit salade. How many different kinds of complete meals can be offered? Problem - given in grade: 6	Reference to mental images

TABLE II

SPONTANEOUS RECOURSE TO A REPRESENTATION

Grade \ Problems variables	Temporal organization		Spatial organization	
	Images	No reference to mental images (cf. Table I, problem 1) for example	Images (cf. Table I, problem 2) for example	No reference to mental images
5e	35%	0%	36%	7%
6e	43%	21%	71%	29%

TABLE III

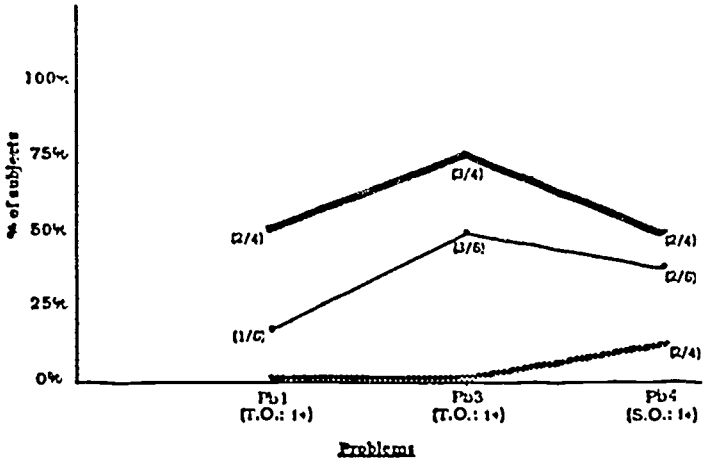
ANALYSIS OF PROGRESS IN RELATION TO PROBLEM SOLVING SKILLS IN 6TH GRADE (5TH GRADE)

Skills \ Number of problems where there was a change Number of subjects	1	2	3
Weak	6/6 (4/6)	4/6 (0/6)	2/6 (0/6)
Average	3/4 (3/4)	1/4 (0/4)	0/4 (0/4)
Able	2/4 (0/4)	1/4 (0/4)	0/4 (0/4)

FIGURE 1a

SPONTANEOUS RECOURSE TO A REPRESENTATION AND PROBLEM SOLVING SKILLS

8th grade



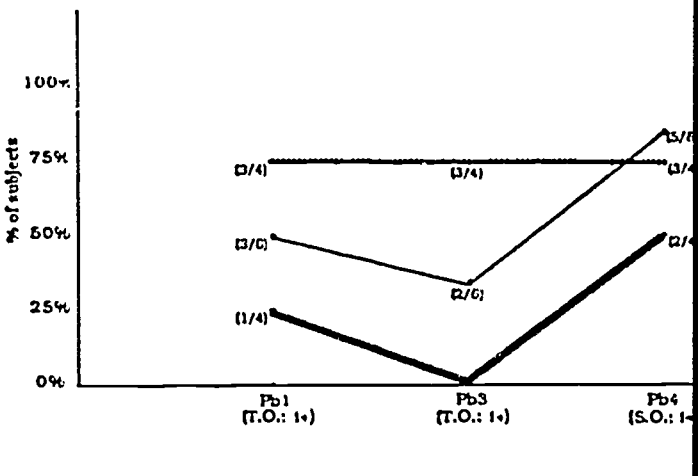
Weak: ——— (/6)
Average: ····· (/4)
Able: ——— (/4)

T.O.: Temporal organization
S.O.: Spatial organization
Pb 1: Window-washers
Pb 3: Gymnastic
Pb 4: Combinations

FIGURE 1b

SPONTANEOUS RECOURSE TO A REPRESENTATI
AND PROBLEM SOLVING SKILLS

8th grade



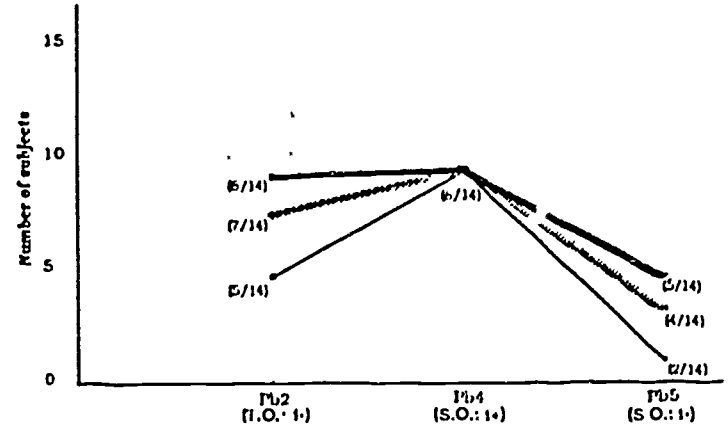
Weak: ——— (/6)
Average: ····· (/4)
Able: ——— (/4)

Pb 1: Window-washers
Pb 3: Gymnastic
Pb 4: Combinations

FIGURE 2a

PERFORMANCE IN WRITTEN TEST, DURING THE INTERVIEW BEFORE REPRESENTATION AND AFTER REPRESENTATION (PROBLEMS IN COMMON TO WRITTEN TEST AND INTERVIEW)

8th grade



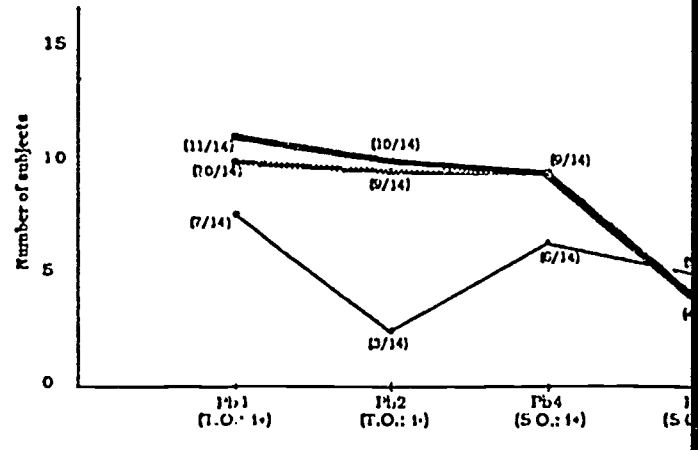
Written test: ———
Solution 1: ·····
Solution 2: ———

1•: Refering to images
1·: Not refering to images
Pb 2: Counters
Pb 4: Combinations
Pb 5: Candles

FIGURE 2b

PERFORMANCE IN WRITTEN TEST, DURING INTERVIEW BEFORE REPRESENTATION AND AFTER REPRESENTATION (PROBLEMS IN COMMON TO WRITTEN TEST AND INTERVIEW)

8th grade



Written test: ———
Solution 1: ·····
Solution 2: ———

Pb 1: Window-washers
Pb 2: Counters
Pb 4: Combinations
Pb 5: Candles

FIGURE 3

REPRESENTATION OF THE STRUCTURE OF THE PROBLEM
AS A FUNCTION OF AGE AND TYPE OF PROBLEM